5. Zh. Zheenbaev, G. A. Kobtsev, R. I. Konavko, and V. S. Éngel'sht, Izv. Sibirsk. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 3, Part 1 (1973).
6. D. I. Vainboim, V. M. Gol'dfarb, and A. D. Yurk, in: Low-Temperature Plasma Generators [in Russian], Énergiya, Moscow (1969), p. 558.
7. A. S. Koroteev and O. I. Yas'ko, Inzh.-Fiz. Zh., 10, No. 1 (1966).
8. A. I. Zhidovich and O. I. Yas'ko, Inzh.-Fiz. Zh., 16, No. 3 (1969).
9. A. G. Shashkov, L. Kreichi, V. I. Krylovich, V. L. Sergeev, F. B. Yurevich, and O. I. Yas'ko, Heat Exchange in an Electric-Arc Gas Heater [in Russian], Énergiya, Moscow (1974).
10. M. F. Zhukov, A. S. Koroteev, and B. A. Uryukov, Applied Dynamics of a Thermal Plasma [in Russian], Nauka, Novosibirsk (1975).
11. M. F. Zhukov and Yu. I. Sukhinin, Izv. Sibirsk. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 3, Part 1 (1969).
12. A. S. An'shakov, M. F. Zhukov, M. I. Sazonov, and A. N. Timoshevskii, Izv. Sibirsk. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 8, Part 2 (1970).
13. G. Yu. Dautov and M. F. Zhukov, Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1965).
14. S. S. Kutateladze and O. I. Yas'ko, Inzh.-Fiz. Zh., 7, No. 4 (1964).
15. A. G. Shashkov, V. L. Sergeev, G. M. Bezladnov, and V. P. Terent'ev, Inzh.-Fiz. Zh., 25, No. 3 (1973).
16. W. Finkel'nburg and H. Mekker, Electric Arcs and Thermal Plasma [Russian translation], Inostr. Lit., Moscow (1961).

SOME PROBLEMS OF CONVECTIVE DIFFUSION TO A SPHERICAL PARTICLE WITH Pe ≤ 1000

B. M. Abramzon and G. A. Fishbein

UDC 532.72

The problem of convective heat and mass exchange during the slow motion of a single drop in a uniform and a shear stream, as well as during the motion of a gas bubble in a power-law liquid, is solved using finite-difference methods.

The determination of the intensity of external heat and mass exchange of a spherical particle under the conditions of axisymmetric streamline flow is connected with the solution of the equation of convective diffusion

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C}{\partial r}\right)+\frac{1}{r^2\sin\Theta}\frac{\partial}{\partial\Theta}\left(\sin\Theta\frac{\partial C}{\partial\Theta}\right)-\frac{\mathrm{Pe}}{2}\left(V_r\frac{\partial C}{\partial r}+\frac{V_\Theta}{r}\frac{\partial C}{\partial\Theta}\right)=0 \quad (1)$$

with the following boundary conditions:

$$C|_{r=1}=1;\quad C|_{r\to\infty}=0. \quad (2)$$

In Eq. (1) the components V_r and V_Θ of the liquid velocity are expressed through the stream function by the equations

$$V_r=-\frac{1}{r^2\sin\Theta}\frac{\partial\Psi}{\partial\Theta};\quad V_\Theta=\frac{1}{r\sin\Theta}\frac{\partial\Psi}{\partial r}.$$

In the present report Eq. (1) is analyzed for several model flows pertaining to cases of slow streamline flow over a particle. Since the Schmidt numbers for real liquids have the order of 10^3, the values of the Peclet number lie in the range of $1 \le \mathrm{Pe} \le 1000$ even for small Reynolds numbers (Re ≤ 1).

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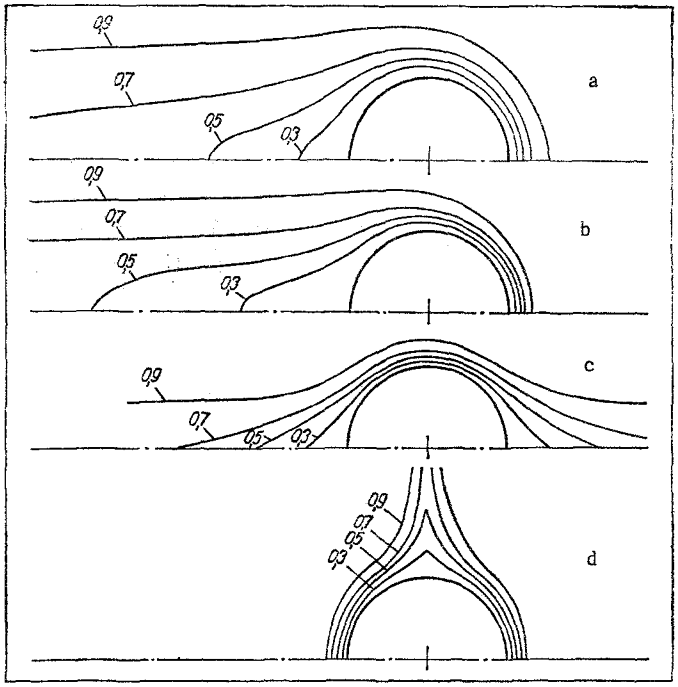

Fig. 1. Concentration field around a spherical particle with Pe = 100 for different models of streamline flow: a) a solid particle in a Stokes stream; b) a gas bubble with Re ≪ 1; c,d) a solid particle in a shear stream (c – $\alpha > 0$; d – $\alpha < 0$).

Analytical solutions of this problem are known in the literature, obtained for the limiting cases of small Peclet numbers by the method of joining of asymptotic expansions [1, 2] and for very large Pe by the method of the diffusional boundary layer [3-9]. A comparison of these approximate solutions with the results of numerical calculations carried out in [10] on the example of the problem of diffusion to a particle in a Stokes flow showed that the method of joining of asymptotic expansions can be used only for Pe ≤ 0.5. As for larger Peclet numbers, the method of the diffusional boundary layer gives acceptable results beginning with Pe ≥ 1000.

The approximate analytical methods prove to be unacceptable in the intermediate region of Peclet numbers. We studied this region of Peclet numbers using the finite-difference method used earlier in [10].

The cases of the mass exchange of a drop during slow motion in a uniform and a shear stream are analyzed. The effect of the non-Newtonian properties of the liquid on the process of mass exchange of a gas bubble is also studied.

The effect of the flow model on the nature of the diffusional interaction of a particle with a stream can be analyzed qualitatively by considering the concentration fields near the sphere (Fig. 1). The process of mass exchange is characterized quantitatively by the values of the local and average Sherwood numbers

$$\mathrm{Sh}_\Theta = -2\left(\frac{\partial C}{\partial r}\right)_{r=1}; \quad \mathrm{Sh} = \frac{1}{2}\int_0^\pi \mathrm{Sh}_\Theta \sin\Theta\, d\Theta. \tag{3}$$

The results of the calculations for the types of flow under consideration are presented in Figs. 2 and 3 in the form of dependences of Sh_Θ and Sh on Pe and other parameters of the problem.

1. Mass Transfer to a Drop with Re ≪ 1

In this case the liquid flow is described by the Hadamard – Rybchinskii stream function

$$\Psi = \frac{1}{2}\left(r^2 - \frac{3\mu+2}{2\mu+2}r + \frac{\mu}{2\mu+2}\frac{1}{r}\right)\sin^2\Theta \tag{4}$$

and the solution of the diffusional problem depends on two parameters: the Peclet number and the viscosity ratio μ.

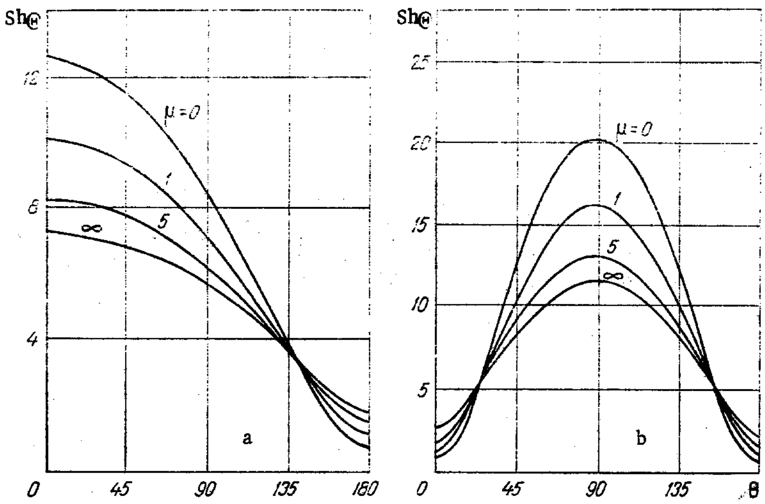

Fig. 2. Local values of Sherwood number with Pe = 100: a) flow over a drop by a uniform stream; b) flow over a drop by a shear stream with $\alpha > 0$.

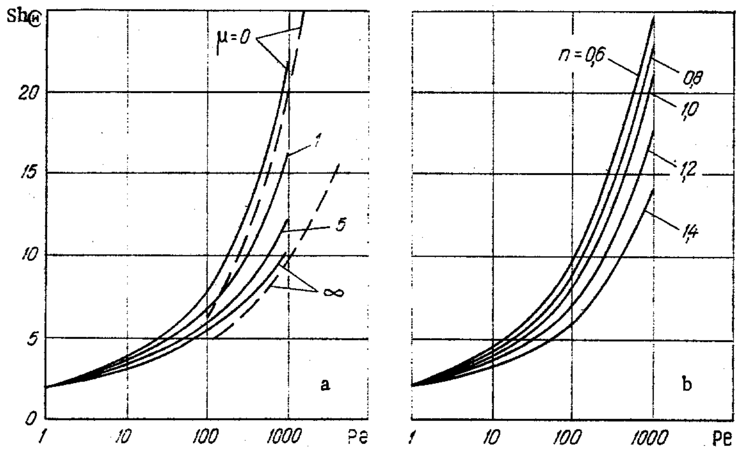

Fig. 3. Dependence of average Sherwood number on Peclet number: a) slow streamline flow over a drop; b) flow over a gas bubble by a stream of power-law liquid.

The dependence $Sh_\Theta(\Theta)$ for Pe = 100 with different μ is presented in Fig. 2a. With the transition from a solid sphere ($\mu = \infty$) to a gas bubble ($\mu = 0$) one observes a considerable increase in the intensity of mass exchange at the frontal part of the sphere and a simultaneous decrease in this value in the rear region. In this case, as seen from Fig. 1a,b, the zone of the main concentration drop narrows in the front part of the sphere and the carrying away of material in the back section is strengthened. This is due to the increase in the role of the convective component of the diffusional flux owing to an increase in the liquid velocity near the surface of the particle with a decrease in μ.

The dependence of the average values of the Sherwood number on the Peclet number for different μ is given in Fig. 3a. The dashed curves are constructed for a solid sphere and a gas bubble from the following equations of boundary-layer theory [3]

$$\mathrm{Sh} = 0.991\,\mathrm{Pe}^{1/3} \quad (\text{for } \mu = \infty), \tag{5}$$

$$\mathrm{Sh} = \frac{0.65}{\sqrt{1+\mu}}\,\mathrm{Pe}^{1/2}. \tag{6}$$

We note that the solutions (5) and (6) give somewhat understated results for the Sherwood number. This is connected with the fact that the method of linearization of the stream function used in [3] leads to understated values of the tangential velocity component near the surface of the particle. The use of a more exact expression for the stream function near the surface of a solid sphere [4] gives the dependence

$$\mathrm{Sh} = 0.992 + 0.991\,\mathrm{Pe}^{1/3}, \tag{7}$$

which is found to be in good agreement with the results of a numerical solution for Pe > 10.

Another defect of the boundary-layer solution (6) for a drop is the impossibility of using it for large μ [5, 6] (a limiting transition to (5) as $\mu \to \infty$ is absent). To find Sh for intermediate values of μ one can use the following approximate equation:

$$\mathrm{Sh}(\mu) = \frac{\mathrm{Sh}(0) + \mu\,\mathrm{Sh}(\infty)}{1+\mu}. \tag{8}$$

Here Sh(0) and Sh(∞) are the values of the Sherwood number with a given Pe for a bubble and a solid sphere.

2. Mass Transfer to a Drop in a Shear Stream

An example of uniform axisymmetric shear flow is the motion of a liquid near a particle which, being fully entrained by the stream, moves along the axis of a convergent (divergent) channel. The stream function for this type of flow around a liquid drop was obtained in Taylor's report [9]:

$$\Psi = \left(r^3 - \frac{5\mu+2}{2\mu+2} + \frac{3\mu}{2\mu+2}\,\frac{1}{r^2}\right)\sin^2\Theta\cos\Theta. \tag{9}$$

In this expression the stream function is written in dimensionless form, with the quantity $U_* = \alpha a$ being used as the characteristic scale of the hydrodynamic velocity. The parameter α determines the intensity and direction of shear. For $\mathrm{Pe} = \alpha a^2/D \gg 1$ the diffusion problem was solved in [7] by the boundary-layer method. The functions

$$\mathrm{Sh} = 2.44\,\mathrm{Pe}^{1/3} \quad (\text{for } \mu = \infty), \tag{10}$$

$$\mathrm{Sh} = \frac{1.95}{\sqrt{1+\mu}}\,\mathrm{Pe}^{1/2} \tag{11}$$

obtained for the Sherwood number are analogous in structure to Eqs. (5) and (6).

The concentration fields around a solid sphere in a shear stream with Pe = 100 and α of different signs are shown in Fig. 1c,d. Figure 1c corresponds to the case of $\alpha > 0$ (motion of the particle along the axis of a convergent channel). Notwithstanding such an important difference in the pattern of the diffusional interaction, the average Sherwood numbers prove to be independent of the sign of α. This fact was established earlier in [7] for Pe ≫ 1. The distribution of local coefficients of mass exchange with Pe = 100 and $\alpha > 0$ is presented in Fig. 2b for different μ. The symmetry of the curves of $\mathrm{Sh}_\Theta(\Theta)$ relative to the plane $\Theta = \pi/2$ follows from Eq. (9) for the stream function.

3. Mass Transfer to a Gas Bubble Moving in a Power-Law Liquid

The stream function for this case

$$\Psi = \frac{1}{2}\left\{(r^2 - r) - \frac{6n(n-1)}{2n+1}\left[r\ln r + \frac{1}{6r} - \frac{r}{6}\right]\right\}\sin^2\Theta \tag{12}$$

was obtained in [8] by the perturbation method and is valid for values of the rheological parameter n not too different from unity. As follows from this expression, the tangential component of the liquid velocity near the surface of the bubble increases with a decrease in n. This leads to the fact that the coefficient of mass exchange proves to be higher for pseudoplastic liquids and lower for dilatant liquids than for Newtonian liquids (see Fig. 3b).

For larger Peclet numbers (Pe ≳ 100-1000) our calculations are in satisfactory agreement with the dependence

$$\mathrm{Sh} = 0.65\left[1 - \frac{4n(n-1)}{2n+1}\right]^{1/2}\sqrt{\mathrm{Pe}}. \tag{13}$$

obtained in the approximation of the theory of a diffusional boundary layer [8].

NOTATION

Pe = $2Ua/D$, Peclet number; Sh_Θ, local value of Sherwood number; Sh, average value of Sherwood number; Re = $2Ua/\nu$, Reynolds number; C, relative mass concentration of transported component; r, radial coordinate (normalized to radius of sphere); Θ, angular coordinate; a, radius of sphere; D, coefficient of diffusion; ν, coefficient of kinematic viscosity; U, velocity of impinging stream; U_*, scale of velocity in shear stream; V_r, V_Θ, radial and tangential velocity components; Ψ, stream function; μ, ratio of coefficients of dynamic viscosity of liquid in drop and of continuous medium; α, parameter characterizing shear intensity; n, rheological parameter of power-law liquid.

LITERATURE CITED

1. A. Acrivos and T. D. Taylor, Phys. Fluids, 5, No. 4, 378-394 (1962).
2. Yu. P. Gupalo and Yu. S. Ryazantsev, Prikl. Mat. Mekh., 35, No. 2 (1971).
3. V. G. Levich, Physicochemical Fluid Dynamics [in Russian], Fizmatgiz, Moscow (1959).
4. A. Acrivos and J. D. Goddard, J. Fluid Mech., 23, Part 2, 273-291 (1965).
5. V. V. Dil'man, Inzh.-Fiz. Zh., 8, No. 3 (1965).
6. Yu. A. Buevich and Yu. A. Korneev, Inzh.-Fiz. Zh., 25, No. 4 (1973).
7. Yu. P. Gupalo and Yu. S. Ryazantsev, Prikl. Mat. Mekh., 36, No. 3 (1972).
8. T. Hirose and M. Moo-Young, Canad. J. Chem. Eng., 47, 265 (1969).
9. G. I. Taylor, Proc. Roy. Soc. A, 138, No. 834, 41-48 (1932).
10. B. M. Abramzon, V. Ya. Rivkind, and G. A. Fishbein, Inzh.-Fiz. Zh., 30, No. 1 (1976).

EFFECT OF COMPRESSIBILITY ON THE HYDRODYNAMICS OF TWO-PHASE FLOWS

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It is shown that the resistance coefficient of a two-phase one-component mixture depends on the Mach number over a wide range of parameters.

It is known that in geometrically similar systems the hydrodynamics of single-phase streams is determined by their compressibility and viscosity.

It has been justified theoretically and confirmed experimentally that the velocity of sound in a two-phase medium with a definite ratio of the phases can be two orders of magnitude smaller than in the liquid phase and more than an order of magnitude smaller than the velocity of sound in the gas. Yet, until recently, calculational models for estimating friction loss in two-phase flow have taken account of the Reynolds number but not the Mach number.

It is shown [1] that a one-component two-phase mixture has the greatest compressibility. Experiments with high-velocity gas flows in long horizontal tubes [2, 3] showed that for Mach numbers greater than 0.75-0.85 the resistance coefficient decreases with increasing Mach number and approaches zero as M approaches unity.

In the present paper we present the dependence of the resistance coefficient on the Mach number for the flow of a two-phase one-component mixture in tubes of constant diameter. We have processed the results of our experiments on the critical outflow of boiling water through long horizontal tubes with a sharp entrance edge. The pressure at the entrance to the experimental section varied from 10^6 to $9.3 \cdot 10^6$ N/m^2; the tube diameters were $14.2 \cdot 10^{-3}$, $9.8 \cdot 10^{-3}$, and $5.8 \cdot 10^{-3}$ m; the relative length was $141 \leq L/D \leq 612$; and the mass flux density varied from $0.567 \cdot 10^4$ to $2.983 \cdot 10^4$ kg/(m$^2 \cdot$ sec).

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